PR-003-1602001

PR-003-1602001 Seat No. ________

M. Phil. (Sem. II) Examination

August - 2020

Mathematics : 18 - CMT - 20001

(Some Topics in Algebraic Topology)

Faculty Code : 003
Subject Code : 1602001

Time : **3** Hours] [Total Marks : **100**

Instructions : (1) All questions are compulsory.
(2) Each question carries 20 marks.

1 Answer following **ten** questions : **10×2=20**

(i) Define Basis for closed sets in a topological space (X, τ).

(ii) Let (X, τ) be a topological space and $\mathcal{C}$ be a basis for closed sets in X. Prove that $\mathcal{B} = \{X - C / C \in \mathcal{C}\}$ is a basis for (X, τ).

(iii) Define term : compact space. Give an example of a topological space is a compact space and give an example of a topological space is not a compact space too.

(iv) Define term : finite intersection property.

(v) Define Filter and Ultra Filter.

(vi) Prove that, every singleton set in $\mathbb{R}$ is zero set of $\mathbb{R}$.

(vii) Define term *Z*–filter.

(viii) Define C^* – embedded and C – embedded.

(ix) Define fixed ideal, free ideal, fixed *Z*–filter and free *Z*–filter.

(x) Define two equivalent compactifications.

2 Answer any **two** questions : **2×10=20**

(1) Let *X* be a topological space and *I* be an ideal of $C(X)$. Prove that $Z(I) = \{Z(f) / f \in I\}$ is a Z – filter on *X*.

(2) Let $\mathcal{F}$ be a Z – filter on *X*. Prove that $Z^{-1}(\mathcal{F}) = \{f \in C(X) / Z(f) \in \mathcal{F}\}$ is an ideal of $C(X)$.

(3) Let $f, g \in C(X)$. Prove that

(1) $Z(f\,g) = Z(f) \cup Z(g)$,

(2) $Z(|f|) = Z(f)$,

(3) $Z(|f|+|g|) = Z(f) \cap Z(g)$,

(4) $Z(f) \cap Z(g) \subseteq Z(f+g)$ and

(5) $Z(f) \cap Z(g) = Z(f^2 + g^2)$.

3 Answer any **one** question : **1×20=20**

(1) Let (K, h) be a compactifications of X and $h(X)$ is C^* –embedded in K. Prove that (K, h) is equivalent to $(\beta X, e)$.

(2) Prove that, every free maximal ideal in $C(X)$ contain the map $j : \mathbb{N} \to \mathbb{R}$ defined by $j(n) = \frac{1}{n}$, for every $n \in \mathbb{N}$.

(3) Let X be a dense subspace of T. Prove that following statements are equivalent :

(1) For a compact Hausdorff space Y, if $f : X \to Y$ is continuous, then there is a continuous map $g : T \to Y$ such that $g(x) = f(x)$, for all x in X.

(2) X is C^* –embedded in T.

(3) If Z_1, Z_2 be two disjoint zero sets in X, then closure of Z_1 in T and closure of Z_2 in T are disjoint.

(4) If Z_1, Z_2 be two zero sets in X, then closure of $Z_1 \cap Z_2$ is equal to intersection of closure of Z_1 in T and closure of Z_2 in T.

(5) There is a unique Z – ultra filter $\mathcal{F}$ on X such that $\mathcal{F} \to p$ in T.

4 Answer any **two** questions : **2×10=20**

(a) Give an example of a C^* –embedded subspace, which is not C – embedded with require justification.

(b) Let X be a space and $p \in X$. Prove that $M_p = \{f \in C(X) / f(p) = 0\}$ is a maximal ideal in $C(X)$.

(c) Let X, Y be two compact spaces. Prove that, X and Y are isomorphic if and only if $C(X), C(Y)$ are isomorphic.

5 Answer any **two** questions : **2×10=20**

(1) Define Z – ideal. Let $\{I_\alpha / \alpha \in J\}$ be a family of Z – ideals in $C(X)$. prove that $\bigcap_{\alpha \in J} I_\alpha$ is also a Z – ideal in $C(X)$.

(2) Let X be a compact Hausdorff space and I be any ideal in $CC(X)$. Prove that the closure of I is also an ideal in $CC(X)$, where $CC(X)$ is the set of all complex valued function on X.

(3) Let I be a Z – ideal in $C(X)$. Does I the Jacobson radical in (X) ? Justify your answer.

(4) Let X be a space and I be an ideal in $C(X)$. Prove that following statements are equivalent :

(i) $Z^{-1}(Z(I)) = I$

(ii) If $f \in C(X)$ and $Z(f) \in Z(I)$, then $f \in I$.

(iii) If $f \in C(X)$ and $Z(f) \in Z(g)$, for some $g \in I$, then $f \in I$.